The role of number notation in numerical processing

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1 The problem with number notations

Number notations can influence the way numbers are handled in computations, however, the role of notation itself in calculations has not been examined directly.

	Sign value notation Roman example: XXIII	Place value notation Indo-Arabic example: 23
Noting the powers (e.g. 1, 10, 100 in a base 10 system)	**Symbol** X means ten I means one	**Position** □_ (left position) means tens _□ (right position) means ones
Noting the quantity within a specific power	**Quantity of symbols** ●●● means three ●● means two	**Symbol** 3 means three 2 means two

- It is believed that place value number notation systems are superior to sign value systems (Chrisomalis, 2010; Ifrah, 1999)
- However, sign value notation might have sufficient efficiency:
  - Sign value notations were common in flourishing cultures, such as the ancient Egypt
  - Simple and efficient calculation procedures are known for sign value notations

2 Artificial number notation paradigm

- Base 4 instead of base 10
- New symbols (e.g. 0-Ł, 1-Ө, 2-Ð, 3-И, 4-Я, 16-Ч)

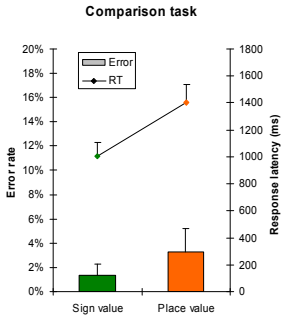
	Sign value notation	Place value notation
Example stimulus	ЯЯӨӨ	ÐИ
The meaning of the stimulus	(4)+(4)+(1)+(1)+(1)=11	(2)*4+(3)*1=11
Example stimulus	ЧЯЯӨӨӨ	ӨÐИ
The meaning of the stimulus	(16)+(4)+(4)+(1)+(1)+(1)=27	(1)*16+(2)*4+(3)*1=27

3 Comparison task

Methods

- Thirty participants
- Compare two multi-power numbers
- In one condition sign value, in other condition place value notations were used
- Procedure:
  - Learn the symbols for the notations
  - Practice comparison until rules are understood
  - Comparison trials, monitoring incorrect rule use

Result



4 Addition task

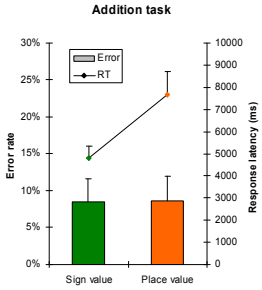
Comparison could be solved with simple tools, e.g. ordering the symbols. Addition requires computationally more complex processing.

	Sign value notation	Place value notation
Example stimulus	ЧӨӨӨ + ЧЧЯЯӨӨ ЧЧЧЯЯЯӨ	ӨЬИ + ÐÐÐ ИИӨ
The meaning of the stimulus	(16)+(1)+(1)+(1) + (16)+(16)+(4)+(4)+(1)+(1) (16)+(16)+(16)+(4)+(4)+(4)+(1)	(1)*16+(0)*4+(3)*1 + (2)*16+(2)*4+(2)*1 (3)*16+(3)*4+(1)*1

Methods

- Eighteen participants
- Check whether the addition is correct
- Procedure:
  - Learn the symbols for the notations
  - Practice addition until rules are understood
  - Addition trials, monitoring incorrect rule use

Result



5 Natural multi-power number representation

Why is sign value number notation easier to process than place value notation?

- Multi-power number representation originally might rely on object and object enumeration representations, forming a **natural multi-power number representation**.
- The structure of sign value notation is more similar to this number representation than the structure of place value notation, thus the transcoding is easier from sign value notation to the number representation.

	Sign value notation	Natural multi-power number representation	Place value notation
Noting the powers	**Symbol** X I	**"Symbol"**	**Position** □_ _□
Noting the quantity within a specific power	**Quantity of symbols** ●●● ●●	**Quantity of symbols** ●●● ●●	**Symbol** 3 2

This model for multi-power numbers can be seen as an extension of McCloskey's abstract model (McCloskey, 1992). The model offers an alternative to the verbal representation and Arabic visual form proposed by Dehaene (1992).

6 Conclusion

- **Sign value notation can be more easily applied** than place value notation for multi-power comparison and addition tasks.
- **Notations have a fundamental effect** of on numerical computations.
- The results are consistent with the **popularity of sign value notations for centuries**.

- We hypothesize a **natural multi-power number representation**.
- A new problem: **why do we use place value notation?** Hypothesis: expertise might change the costs and benefits.

7 References

Chrisomalis, S. (2010). Numerical Notation: A Comparative History (1st ed.). Cambridge University Press.
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Ifrah, G. (1999). The Universal History of Numbers: From Prehistory to the Invention of the Computer (1st ed.). Wiley.
McCloskey, M. (1992). Cognitive mechanisms in numerical processing: evidence from acquired dyscalculia. Cognition, 44(1-2), 107-157.